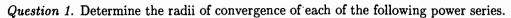
Math 226 Midterm # 2



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Fully answer all the questions in the booklets provided. Cle No communication of any kind with your fellow students. single $8\frac{1}{2} \times 11$ hand written sheet. Time: 2 hours.





(a)
$$\sum_{n=1}^{\infty} x^n \sin\left(\frac{1}{2^n}\right)$$

(b)
$$\sum_{n=1}^{\infty} n!(x-5)^n$$
(d)
$$\sum_{n=1}^{\infty} \frac{x^n}{3^n \sqrt{n}}$$

(c)
$$\sum_{\substack{n=1\\\infty\\\infty\\-2n}}^{n=1} \frac{x^n}{(n!)^3}$$

(d)
$$\sum_{n=1}^{\infty} \frac{x^n}{3^n \sqrt{n}}$$

(e)
$$\sum_{n=1}^{\infty} \frac{x^{2n}}{4^n n^2}$$

Question 2. Determine the interval of convergence of each of the following power series.

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$$(a) \sum_{n=1}^{\infty} \sqrt{n} x^n$$

$$(b) \sum_{n=1}^{\infty} \frac{x^{2n}}{n^2}$$

(a)
$$\sum_{n=1}^{\infty} \sqrt{n}x^n$$
 (b) $\sum_{n=1}^{\infty} \frac{x^{2n}}{n^2}$ (c) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{2^n (n+3)}$

Question 3. Suppose f(x) is a function which has Taylor series

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$$f(x) = 2x - \frac{1}{2}x^2 + 5x^3 + \cdots$$

Calculate, up to and including terms of order x^3 , the Taylor expansions of the functions

(a)
$$f(x) \sin x$$

(b)
$$f(x) + 2\sqrt{1-x}$$

(c)
$$xf'(x)$$
.

Question 4. Find the Taylor series, up to and including terms of order x^2 , of the function

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$$y=\ln(1+2\sin x).$$

Question 5. Find the general solution to the following differential equations.

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$$(a) \quad \frac{dy}{dx} = \frac{y(x^2 - 4)}{x}$$

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$$\frac{dy}{dx} = \frac{y(x^2 - 4)}{x}$$
 (b) $\frac{1}{t} \frac{dy}{dt} - y + 2 = 0$.

Question 6. Find the general solution to the following differential equations.

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(a)
$$\frac{dy}{dx} + \frac{4x}{1+x^2}y = \frac{4x}{1+x^2}$$

(b)
$$x\frac{dy}{dx} + (1+x)y - 1 = 0.$$

Question. 7. Solve the following initial value problems.

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(a)
$$\frac{dy}{dx} + y = x^2$$
, $y(1) = 2$

(b)
$$\frac{dy}{dt} = \frac{y-1}{t-1}$$
, $y(3) = 2$.